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Reliability Analysis with Triangular Hesitant Fuzzy Pareto Life Distribution

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Abstract

Recently, a unique extension of fuzzy sets known as hesitant fuzzy sets has been established to address hesitant cases that previous methods were unable to manage adequately. In this paper, the triangular hesitant fuzzy sets approach has been employed to explore the inherent uncertainty within the parameters of the life distribution. Two essential reliability measures, triangular hesitant fuzzy reliability and the hazard rate function designed for the Pareto Type I life distribution, have been established. Moreover, the triangular hesitant fuzzy reliability measure is utilized to assess the reliability of series and parallel systems. Furthermore, the weighted averaging operator has been used on both the series and parallel systems, making them more reliable and giving much better results than hesitant fuzzy sets. Finally, a numerical example demonstrating the use of these techniques is provided, and the results are presented in tabular and graphical formats.

Keywords: Hesitant fuzzy sets, Triangular fuzzy numbers, Hesitant fuzzy reliability, Pareto Type I distribution, weighted averaging operator.

1 | Introduction

In all aspects, reliability analysis becomes a fact-finding exercise that focuses on evaluating the performance and durability expected from a system, product, or process for use in engineering, healthcare, and financial domains. It is defined as a measure of the probability that a system or component will perform its required function without failing for a specified period of time. Imprecision and vagueness almost always accompany realistic evaluations of reliability due to measurement errors, uncertain data, and the very limited nature of classical probability theory. Zadeh [1] introduced the concept of fuzzy set theory to define and govern

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situations based on fuzzy information. Subsequent to extensive study, other expansions of fuzzy logic have been developed, such as Type 1, Type 2, rough, and Intuitionistic Fuzzy Sets (IFSs) [2].

The Hesitant Fuzzy Set (HFS) [3], which is an expansion of the conventional fuzzy set, can be employed to measure the level of uncertainty in decision-making. It gives a more subtle way to capture the preferences and assessments made by experts, modeling often imperfect and vague real-world data [4], [5]. Since its introduction, the HFS has drawn more attention and has been successfully applied to resolve a range of complex decision-making scenarios [6–10]. For a more precise assessment, HFSs utilize aggregation operators to consolidate several membership values into a singular value, which enhances precision and facilitates an accurate evaluation of the problem. Xia and Xu [11] and Xu [12] developed a variety of hesitant fuzzy aggregation operators, such as hesitant fuzzy weighted, geometric, and averaging operators.

In addition to HFSs, researchers have discovered several other fuzzy extensions, such as Pythagorean, Fermatean, and q-rung picture fuzzy sets. These extensions also play a crucial role in dealing with decision-making problems. Yager introduced the Pythagorean fuzzy method [13] and, later, applied by Akram et al. [14] to study linear programming problems in the context of fuzzy. Furthermore, he utilized the Fermatean [15] for transportation problems and the q-rung picture fuzzy method [16] for multi-criteria decision-making. An additional expansion is the HyperSoft set, which is intended for application in scenarios involving decision-making where multi-attribute functions and more nuanced degrees of uncertainty need to be evaluated [17]. Furthermore, the HyperSoft set is extended to the SuperHyperSoft set [18].

Out of all these expansions, the Neutrosophic set is one of the most beneficial extensions of fuzzy sets, particularly helpful in complex engineering and decision-making situations. Numerous authors have developed and applied the Neutrosophic set [19] to study different types of decisions [20–22] and medical science problems [23]. The extended fuzzy methods are not limited to only decision-making; they have also explored various fields, including supply chain [24], [25], stock trading [26], sustainable development [27], and quality control [28].

In reliability analysis, HFSs offer the flexibility to model complex reliability characteristics, adapt to uncertain environments, and make informed decisions despite imperfect information. For example, Chand et al. [29] employed hesitant and dual Hesitant Fuzzy Elements (HFE) to calculate the reliability of the optical fiber communication system. Also, an advanced probabilistic hesitant fuzzy technique has been presented by Liu et al. [30]. For the evaluation of Satellite Communication System (SCS) capabilities. System reliability relies heavily on maintenance and failure management, which serve an essential role in ensuring uninterrupted and effective operation by proactively eliminating unforeseen periods of inactivity. The incorporation of uncertainty and environmental factors into maintenance plan selection, through the use of a fuzzy approach, improves sustainability and results in more resilient and sustainable systems [31–34].

The Pareto distribution is commonly used to represent reliability and heavy-tailed lifespan data [35], and it has found applications in various scientific study disciplines [36–39]. For system reliability, Roohanizadeh et al. Roohanizadeh et al. [40] and Kalam et al. [41], [42] have studied Pareto as well as other life distributions using generalized IFSs. The current body of research strongly highlights the importance of investigating fuzzy reliability for lifetime distribution, encompassing practical as well as theoretical implications. Therefore, this study focuses on developing a triangular hesitant fuzzy method for Pareto type I life distribution and further applying it to investigate the system's reliability.

The subsequent sections of the paper are arranged as follow. Section 2 provides fundamental definitions related to the theory of fuzzy sets. Furthermore, Section 3 focuses on developing reliability measures, such as reliability and hazard functions, designed for the Pareto I distribution. Then, the developed methods are used to define the mathematical formulations for the reliability of series and parallel systems in Section 4. Finally, Section 5 presents a numerical instance along with an illustration to demonstrate the theoretical concepts.

2 | Preliminaries

2.1 | Fuzzy Set

A fuzzy set $\overline{B}$ in a universe of discourse Y is defined as

$$\overline{B} = \{\langle y, \mu_{\overline{B}}(y)\rangle y \in Y\}. \quad (1)$$

Which is described by a membership function $\mu_{\overline{B}}: Y \to [0,1]$ where, $\mu_{\overline{B}}$ denotes the degree of membership of the element y to the set $\overline{B}$.

2.2 | Fuzzy Number

A fuzzy number, $\overline{B}$ is a fuzzy set on a real number R, holds the given conditions.

I. $\mu_{\overline{B}}(y)$ is piecewise continuous.

II. $\overline{B}$ must be normal (there is at least one $y \in R$ such that, $\mu_{\overline{B}}(y) = 1$.

III. $\overline{B}$must be convex for any $y_1, y_2 \in Y$ and $\mu_{\overline{B}}(\pi y_1 + (1-\pi)y_2) \geq \min\left(\mu_{\overline{B}}(y_1), \mu_{\overline{B}}(y_2)\right)$, where $0 \leq \pi \leq 1$.

2.3 | Intuitionistic Fuzzy Set

Atanassov's IFS $\overline{B}$ on a reference set, Y can be defined as

$$\overline{B} = \{\langle y, \mu_{\overline{B}}(y), \gamma_{\overline{B}}(y)\rangle y \in Y\}, \quad (2)$$

where the $\mu_{\overline{B}}(y)$ and $\gamma_{\overline{B}}(y)$ are the membership and non-membership degrees of the element $y \in Y$ to the set $\overline{B}$, respectively. Also, satisfying the relation $0 \leq \mu_{\overline{B}}(y) + \gamma_{\overline{B}}(y) \leq 1$for each element $y \in Y$.

2.4 | Hesitant Fuzzy Set

A HFS that exists within the universe of discourse Y is described as a function that, when applied to Y, returns a subset of [0, 1].

Torra [3] introduced the above definition of HFS, and Xia and Xu [11] completed it in its mathematical form

$$\overline{B} = \{\langle y, \hbar_{\overline{B}}(y)\rangle \mid y \in Y\}, \quad (3)$$

where $\hbar_{\overline{B}}(y)$ is possible membership degrees of $y \in Y$ to the set $\overline{B}$. For more clarity, $\hbar = \hbar_{\overline{B}}(y)$ is a HFE, and $\Omega = \bigcup \hbar_{\overline{B}}(y)$ is a collection of all HFEs of $\overline{B}$.

Example 1. Suppose that $Y = \{y_1, y_2, y_3\}$ is a fixed set, $\hbar_{\overline{B}}(y_1) = \{0.1, 0.2, 0.3\}$, $\hbar_{\overline{B}}(y_2) = \{0.4, 0.5\}$ and $\hbar_{\overline{B}}(y_2) = \{0.3, 0.5, 0.2, 0.6\}$ are HFEs of $y_i, (i = 1,2,3)$to a set $\overline{B}$, respectively. Then $\overline{B}$, as a HFS.

$$\overline{B} = \{\langle y_1, \{0.1, 0.2, 0.3\}\rangle, \langle y_2, \{0.4, 0.5\}\rangle, \langle y_3, \{0.3, 0.5, 0.2, 0.6\}\rangle\}.$$

Some operations on HFEs:

I. The lower bound of an HFE, $\hbar^- = \min\{\xi \mid \xi \in \hbar\}$.

II. The upper bound of an HFE, $\hbar^+ = \max\{\xi \mid \xi \in \hbar\}$.

III. Complement, $\hbar^c = \bigcup_{\xi \in \hbar}\{1-\xi\}$.

IV. Union of two HFEs, $\hbar_1 \cup \hbar_2 = \bigcup_{\xi_1 \in \hbar_1, \xi_2 \in \hbar_2} \max\{\xi_1, \xi_2\}$.

V. Intersection, $\hbar_1 \cap \hbar_2 = \bigcap_{\xi_1 \in \hbar_1, \xi_2 \in \hbar_2} \min\{\xi_1, \xi_2\}$. .

2.5 | Relation between IFS and HFS

If an IFS is given as $\bar{B} = \{y, \mu_{\bar{B}}(y), \gamma_{\bar{B}}(y)\}$, then the corresponding HFS can be derived as $\hbar_{\bar{B}}(y) = [\mu_{\bar{B}}(y), 1-\gamma_{\bar{B}}(y)]$ if $\mu_{\bar{B}}(y) \neq 1-\gamma_{\bar{B}}(y)$. However, deriving an IFN from HFE is challenging when the HFS comprises multiple elements for each $y \in Y$. To deal with this situation, Torra [3] suggested the envelope of a HFE, which effectively transforms it into an Intuitionistic Fuzzy Number (IFN).

Let $\hbar$ be an HFE; the IFN $\bar{B}_{env}(\hbar)$ is defined as the envelope of $\hbar$, where $\bar{B}_{env}(\hbar) = (\hbar^-, 1-\hbar^+)$, with $\hbar^- = \min\{\xi \mid \xi \in \hbar\}$ and, $\hbar^+ = \max\{\xi \mid \xi \in \hbar\}$ as the lower and upper bounds, respectively.

2.6 | Triangular Hesitant Fuzzy Set

A Triangular Fuzzy Number (TFN) is defined by Kalam et al. [38] as triplet $\bar{B} = \{\eta_1, \eta_2, \eta_3\}$ where η_1, η_2, η_3 are lower, upper and middle values of the fuzzy number and its membership function $\mu_{\bar{B}}: Y \to [0,1]$ is equivalent to

$$\mu_{\bar{B}}(y) = \begin{cases} \dfrac{y-\eta_1}{\eta_2-\eta_1}, \eta_1 \leq y \leq \eta_2, \\ \dfrac{y-\eta_3}{\eta_2-\eta_3}, \eta_1 \leq y \leq \eta_2, \\ 0, \qquad \text{o.w.} \end{cases} \tag{4}$$

Some operations of the TFNs the following:

Let $\bar{B} = \{\eta_1, \eta_2, \eta_3\}$ and $\bar{C} = \{\gamma_1, \gamma_2, \gamma_3\}$ are two TFNs, then:

I. $\bar{B} \oplus \bar{C} = \{\eta_1, \eta_2, \eta_3\} \oplus \{\gamma_1, \gamma_2, \gamma_3\} = \{\eta_1+\gamma_1, \eta_2+\gamma_2, \eta_3+\gamma_3\}$.

II. $\bar{B} \otimes \bar{C} = \{\eta_1, \eta_2, \eta_3\} \otimes \{\gamma_1, \gamma_2, \gamma_3\} = \{\eta_1\gamma_1, \eta_2\gamma_2, \eta_3\gamma_3\}$.

III. $\bar{B} = \bar{C} = \eta_1 = \gamma_1, \eta_2 = \gamma_2, \eta_3 = \gamma_3$.

IV. $\bar{B} \leq \bar{C} = \eta_1 \leq \gamma_1, \eta_2 \leq \gamma_2, \eta_3 \leq \gamma_3$.

V. $\beta.\bar{B} = \{\beta, \beta, \beta\}\{\eta_1, \eta_2, \eta_3\} = \{\beta\eta_1, \beta\eta_2, \beta\,\eta_3\}, \beta \in R > 0$.

VI. $\bar{B}^{-1} = \{\eta_1, \eta_2, \eta_3\}^{-1} = \{1/\eta_1, 1/\eta_2, 1/\eta_3\}$.

Note that, based on the HFS and TFN, if $\hbar = (0,0,0)$ then $\hbar$ is known as a Zero triangular hesitant fuzzy number, and if $\hbar = (1,1,1)$ then $\hbar$ is a Unit triangular hesitant fuzzy number.

2.7 | α-Cut of HFS

If $\bar{B}$ in Y is a fuzzy set and any real number $\alpha \in [0, 1]$, then the α -cut of $\bar{B}$, denoted by $\bar{B}^{\alpha}$ is the crisp set

$$\bar{B}^{\alpha} = \{y \in Y : \mu_{\bar{B}}(y) \geq \alpha\}. \tag{5}$$

To be specific, α-cut of a TFN $\bar{B} = \{\eta_1, \eta_2, \eta_3\}$ can be simplified as

$$\bar{B}^{\alpha} = \{\eta_1 + \alpha(\eta_2 - \eta_1),\ \ \eta_3 - \alpha(\eta_3 - \eta_2)\}. \tag{6}$$

2.8 | Aggregation Operator

Weighted Average Operator (WAO), which is the most popular aggregation operator used in reliability studies, can be defined as n-dimensional mapping $R^n \to R$ with associated weighting $w_j \in [0,1]$.

Suppose, $w = (w_1, w_2, ..., w_n)$ be the associated weighting vector of a triangular hesitant fuzzy variable $h = (h_1, h_2, ..., h_n)$ with $\sum_{j=1}^{n} w_j = 1$. Then, $WOA(h_1, h_2, ..., h_n) = \sum_{j=1}^{n} w_j h_j$ is equivalent to:

$$\bigotimes_{j=1}^{n}(w_j h_j) = \left\{\left(1 - \prod_{j=1}^{n}(1-\eta_1)^{w_j}\right), \left(1 - \prod_{j=1}^{n}(1-\eta_2)^{w_j}\right), \left(1 - \prod_{j=1}^{n}(1-\eta_3)^{w_j}\right)\right\}. \tag{7}$$

3 | Hesitant Fuzzy Reliability Characteristics

Hesitant fuzzy numbers allow analysts to take into consideration degrees of hesitation and uncertainty in the values of crucial parameters in addition to variability. They are distinguished by their triangle representation, which consists of lower, central, and upper bounds. This method makes reliability modeling more thorough and adaptable, especially in situations where reliability data may be imprecise and cannot be sufficiently captured by crisp values.

Consider the Pareto Type I life distribution, where the uncertainty in the shape parameter is expressed by fuzzifying the parameter values into a triangular hesitant fuzzy number. If the lifetime random variable Y with scale parameter φ is and ω as shape parameters, having the probability density function

$$f(y, \omega) = \frac{\omega \varphi^{\omega}}{y^{\omega+1}}, \quad y > \varphi,\ \omega > 0,\ \varphi > 0.$$

the CDF

$$F_Y(y) = 1 - \left(\frac{\varphi}{y}\right)^{\omega}, \quad y \geq \varphi.$$

and the reliability function is

$$S(t) = \left(\frac{\varphi}{t}\right)^{\omega}.$$

Since $\left(\frac{\varphi}{t}\right)^{\omega}$ is a decreasing function with ω, the $\alpha-$cut of the hesitant fuzzy reliability function will be derived as

$$S(t) = p(y > t) = 1 - F_Y(t) = \left[1 - F_{max}(y)[\alpha], 1 - F_{min}(y)[\alpha], \mu_{F(y)} = \alpha\right],\ \ t > 0.$$

or,

$$S(t)[\alpha] = \left[S(t)^L[\alpha], S(t)^R[\alpha]\right] = \left[\left(\frac{\varphi}{t}\right)^{\eta_3 - \alpha(\eta_3 - \eta_2)}, \left(\frac{\varphi}{t}\right)^{\eta_1 + \alpha(\eta_2 - \eta_1)}\right], \tag{8}$$

where,

$$S^L(t)[\alpha] = \inf_{\omega \in \omega[\alpha]} S(t),\ \ S^R(t)[\alpha] = \sup_{\omega \in \omega[\alpha]} S(t).$$

The hazard function is given as

$$h(t)=\left(\frac{\omega}{t}\right).$$

Since $\left(\frac{\omega}{t}\right)$ is increasing function with ω, the $\alpha-$cut of the hesitant fuzzy reliability function will be calculated as

$$h(t)[\alpha]=\left[h(t)^{L}[\alpha],h(t)^{R}[\alpha]\right]=\left[\left(\frac{\eta_3-\alpha(\eta_3-\eta_2)}{t}\right),\left(\frac{\eta_1+\alpha(\eta_2-\eta_1)}{t}\right)\right], \quad \textbf{(9)}$$

where, $h^{L}(t)[\alpha]=\inf\limits_{\omega\in\omega[\alpha]} h(t),\quad h^{R}(t)[\alpha]=\sup\limits_{\omega\in\omega[\alpha]} h(t).$

4 | Hesitant Fuzzy Reliability of Systems

4.1 | Reliability of a Series System

A series system is one in which each unit or component is responsible for the system's overall reliability, and the system fails if even one of those components does. Let's assume that a series system interconnects k units, and the component lifetimes conform to the Pareto Type I distribution. Using the concept of Triangular Hesitant Fuzzy Set (THFS), we can calculate the *a*-cut of the reliability function for a series system as:

$$S_{sr}(t)[\alpha]=\left[P\left(Y_k\le t\,|\,\omega\in\omega[\alpha]\right)\right]=\left[\left(S(t)\right)^{k}\,|\,\omega\in\omega[\alpha]\right]=\left[S^{L}(t)[\alpha],S^{R}(t)[\alpha]\right]=$$
$$\left[\left(\frac{\varphi}{t}\right)^{k\left(\eta_3-(\eta_3-\eta_2)\alpha\right)},\left(\frac{\varphi}{t}\right)^{k\left(\eta_1+(\eta_2-\eta_1)\alpha\right)}\right], \quad \textbf{(10)}$$

where, $S^{L}(t)[\alpha]=\inf\limits_{\omega\in\omega[\alpha]}\left(S(t)\right)^{k},\quad S^{R}(t)[\alpha]=\sup\limits_{\omega\in\omega[\alpha]}\left(S(t)\right)^{k}.$

4.2 | Reliability of a Parallel System

A parallel system is a system that remains fully operational as long as at least one of its components is functional. The system experiences failure only when all of its components fail simultaneously. If the component lifetimes follow the Pareto Type I distribution, the *a*-cut of the reliability function for a parallel system is as follows:

$$S_{pr}(t)[\alpha]=\left[P\left(Y_k>t\,|\,\omega\in\omega[\alpha]\right)\right]=\left[1-\left(1-S(t)\right)^{k}\,|\,\omega\in\omega[\alpha]\right]$$
$$=\left[S^{L}(t)[\alpha],S^{R}(t)[\alpha]\right]=\left[\left\{1-\left(1-\left(\frac{\varphi}{t}\right)^{(\eta_3-(\eta_3-\eta_2)\alpha}\right)^{k}\right\},\left\{1-\left(1-\left(\frac{\varphi}{t}\right)^{(\eta_1+(\eta_2-\eta_1)\alpha}\right)^{k}\right\}\right], \quad \textbf{(11)}$$

where, $S^{L}(t)[\alpha]=\inf\limits_{\omega\in\omega[\alpha]}\left(1-\left(1-S(t)\right)^{k}\right),\quad S^{R}(t)[\alpha]=\sup\limits_{\omega\in\omega[\alpha]}\left(1-\left(1-S(t)\right)^{k}\right).$

5 | Numerical Example

Let a Pareto Type I distribution with a hesitant membership function be used to model the lifetime of an electronic unit in a system. If $\phi_1:\left(\eta_1=0.25,\eta 2=0.35,\eta_3=0.45\right)$, $\phi_2:\left(\eta_1=0.40,\eta_2=0.55,\eta_3=0.65\right)$ and $\phi_3:\left(\eta_1=0.51,\eta_2=0.60,\eta_3=0.69\right)$ are three triangular hesitant fuzzy numbers, and equal weight $1/3$ is assigned to each member, then the *a*-cut of the membership function for the three triangular hesitant fuzzy numbers is given as

$$\begin{cases} \phi_1 = (0.25+0.1\alpha, 0.45-0.1\alpha), \\ \phi_2 = (0.4+0.15\alpha, 0.65-0.1\alpha), \\ \phi_3 = (0.51+0.09\alpha, 0.69-0.09\alpha). \end{cases} \quad (12)$$

Using *Eq. (8)* and *Eq. (12)*, the $\alpha-$cut of the hesitant fuzzy reliability function for $\varphi = 1$ is given by

$$S(t)[\alpha] = \begin{cases} \left(\frac{1}{t}\right)^{(0.45-0.1\alpha)}, \left(\frac{1}{t}\right)^{(0.25+0.1\alpha)}, \\ \left(\frac{1}{t}\right)^{(0.65-0.1\alpha)}, \left(\frac{1}{t}\right)^{(0.4+0.15\alpha)}, \\ \left(\frac{1}{t}\right)^{(0.69-0.09\alpha)}, \left(\frac{1}{t}\right)^{(0.51+0.09\alpha)}. \end{cases} \quad (13)$$

As a means to enhance clarity, we employed *Eq. (13)* to visually depict the fuzzy reliability of the three hesitant TFNs in *Fig. 1*.

This graphical representation demonstrates that the third number $\phi_3 : (\eta_1 = 0.51, \eta_2 = 0.60, \eta_3 = 0.69)$ significantly outperforms the other two in terms of reducing uncertainty, as evidenced by its superior reliability bands.

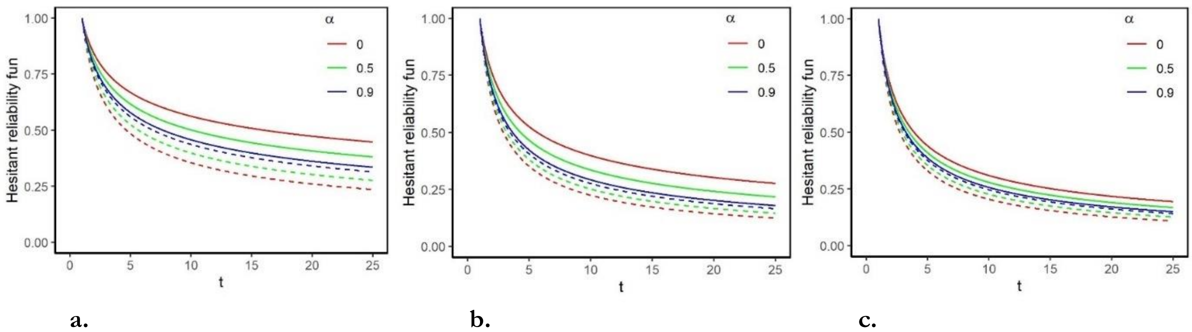


Fig. 1. The $\alpha-$cut for the hesitant fuzzy reliability function; a. $\boldsymbol{\phi_1 = (0.25, 0.35, 0.45)}$, b. $\boldsymbol{\phi_2 = (0.40, 0.55, 0.65)}$, c. $\boldsymbol{\phi_1 = (0.51, 0.60, 0.69)}$.

Analogously, the other reliability measure, hesitant fuzzy hazard, is based on *Eq. (9)* and *Eq. (12)* in the form of their $\alpha-$cut for $\varphi = 1$ as follows:

$$h(t)[\alpha] = \begin{cases} \left(\frac{(0.45-0.1\alpha)}{t}\right), \left(\frac{(0.25+0.1\alpha)}{t}\right), \\ \left(\frac{(0.65-0.1\alpha)}{t}\right), \left(\frac{(0.4+0.15\alpha)}{t}\right), \\ \left(\frac{(0.69-0.09\alpha)}{t}\right), \left(\frac{(0.51+0.09\alpha)}{t}\right). \end{cases} \quad (14)$$

Also, the graphical representation of hazard bands in *Fig. 2* illustrates that the third triangular hesitant fuzzy number significantly outperforms in terms of enhancing precision.

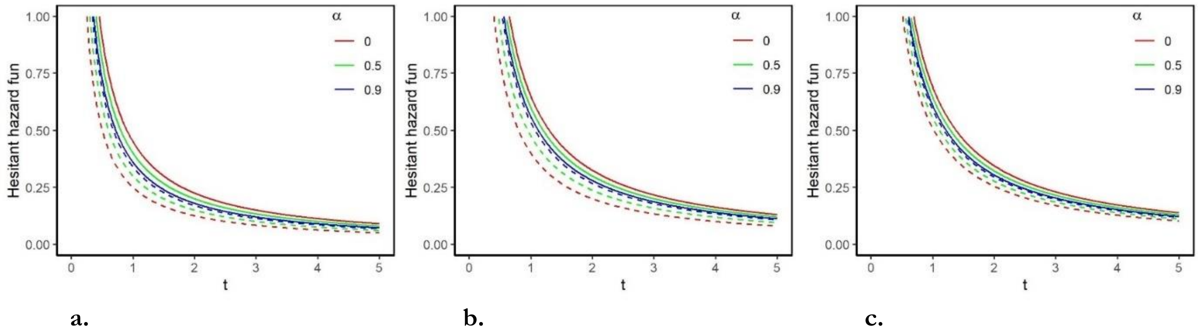


Fig. 2. The α-cut for the hesitant fuzzy hazard function; a. $\phi_1 = (0.25, 0.35, 0.45)$, b. $\phi_2 = (0.40, 0.55, 0.65)$, c. $\phi_1 = (0.51, 0.60, 0.69)$.

Taking $\varphi = 1$, the reliability function for the series system consisting of $k = 5$ components:

$$S_{sr}(t)[\alpha] = \left[\begin{array}{l} \left(\dfrac{1}{t}\right)^{5(0.45-0.1\alpha)}, \left(\dfrac{1}{t}\right)^{5(0.25+0.1\alpha)}, \\ \left(\dfrac{1}{t}\right)^{5(0.65-0.1\alpha)}, \left(\dfrac{1}{t}\right)^{5(0.4+0.15\alpha)}, \\ \left(\dfrac{1}{t}\right)^{5(0.69-0.09\alpha)}, \left(\dfrac{1}{t}\right)^{5(0.51+0.09\alpha)}. \end{array}\right. \quad (15)$$

Using *Eq. (15)*, the $\alpha-$cut of reliability functions for the series system at time $t = 5$ is numerically calculated and provided in *Table 1*.

Table 1. The $\alpha-$cut of reliability functions for the series system.

α	$S_{sr1}[\alpha]$	$S_{sr2}[\alpha]$	$S_{sr3}[\alpha]$
0	0.0267, 0.1337	0.0053, 0.0400	0.0039, 0.0165
0.1	0.0290, 0.1234	0.0058, 0.0355	0.0042, 0.0154
0.3	0.0341, 0.1051	0.0068, 0.0278	0.0048, 0.0133
0.5	0.0400, 0.0894	0.0080, 0.0219	0.0056, 0.0115
0.7	0.0470, 0.0761	0.0094, 0.0172	0.0064, 0.0099
0.9	0.0552, 0.0648	0.0110, 0.0135	0.0074, 0.0086
1	0.0598, 0.0598	0.0120, 0.0120	0.0080, 0.0080

The reliability of the series system is presented in *Table 1* and depicted graphically in *Fig. 3*.

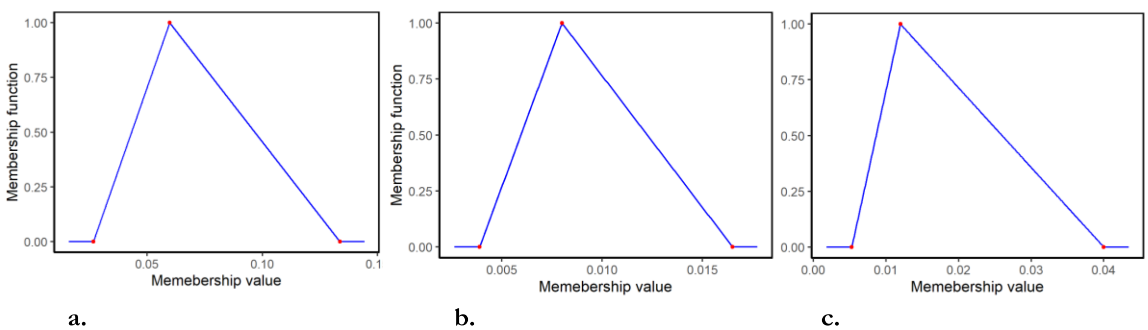


Fig. 3. The $\alpha-$cut of reliability function for the series system; a. $\phi_1 = (0.25, 0.35, 0.45)$, b. $\phi_2 = (0.40, 0.55, 0.65)$, c. $\phi_1 = (0.51, 0.60, 0.69)$.

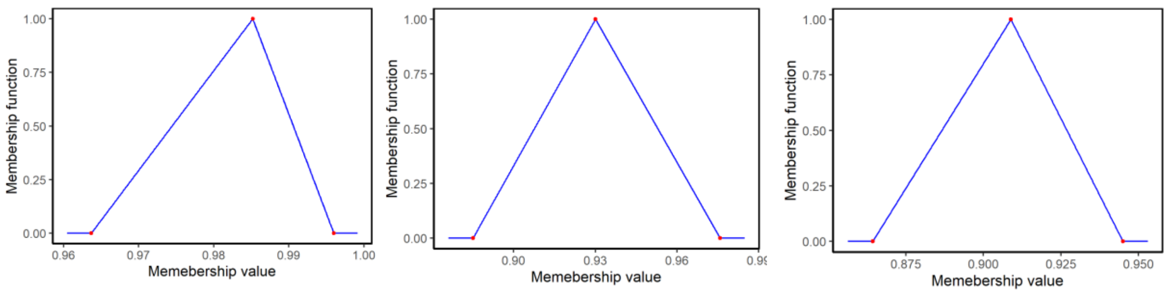
In the same way, the $\alpha-$cut of the reliability function of the parallel system for the parameter value $\varphi=1$ and $t=5$ is given by the following and numerically presented in *Table 2*.

$$S_{pr}(t)[\alpha]=\left[\begin{array}{l}\left\{1-\left(1-\left(\frac{1}{t}\right)^{(0.45-0.1\alpha)}\right)^5,\left\{1-\left(1-\left(\frac{1}{t}\right)^{(0.25+0.1\alpha)}\right)^5\right\}\right\},\\ \left\{1-\left(1-\left(\frac{1}{t}\right)^{(0.65-0.1\alpha)}\right)^5,\left\{1-\left(1-\left(\frac{1}{t}\right)^{(0.4+0.15\alpha)}\right)^5\right\}\right\},\\ \left\{1-\left(1-\left(\frac{1}{t}\right)^{(0.69-0.09\alpha)}\right)^5,\left\{1-\left(1-\left(\frac{1}{t}\right)^{(0.51+0.09\alpha)}\right)^5\right\}\right\}.\end{array}\right. \tag{16}$$

Table 2. The cut of reliability functions for the parallel system.

α	$S_{pr1}[\alpha]$	$S_{pr2}[\alpha]$	$S_{pr3}[\alpha]$
0	0.9637, 0.9960	0.8851, 0.9759	0.8644, 0.9450
0.1	0.9664, 0.9953	0.8901, 0.9725	0.8692, 0.9418
0.3	0.9714, 0.9937	0.8997, 0.9650	0.8785, 0.9351
0.5	0.9759, 0.9918	0.9089, 0.9564	0.8876, 0.9281
0.7	0.9800, 0.9894	0.9177, 0.9467	0.8964, 0.9207
0.9	0.9836, 0.9867	0.9261, 0.9359	0.9048, 0.9129
1	0.9852, 0.9852	0.9301, 0.9301	0.9089, 0.9089

Calculated parallel system reliabilities in *Table 2* are also shown graphically in *Fig. 4*.



a. **b.** **c.**

Fig. 4. The $\alpha-$cut of reliability function for the parallel system; a. $\phi_1=(0.25,0.35,0.45)$, b. $\phi_2=(0.40,0.55,0.65)$, c. $\phi_1=(0.51,0.60,0.69)$.

Weighted averaging operators (*Eq.(7)*) are mathematical functions that reduce a set of values to a single meaningful value. Now, allocating equal weight to every lower and upper cut value in *Table 1*, *Table 2*, and presenting the aggregated reliability in *Table 3*.

Table 3. Reliability of series and parallel systems after aggregation.

α	$S_{asr}[\alpha]$	$S_{apr}[\alpha]$
0	0.0120, 0.0648	0.9173, 0.9826
0.1	0.0131, 0.0593	0.9215, 0.9804
0.3	0.0153, 0.0496	0.9296, 0.9757
0.5	0.0180, 0.0416	0.9373, 0.9705
0.7	0.0211, 0.0349	0.9445, 0.9645
0.9	0.0248, 0.0293	0.9513, 0.9580
1	0.0269, 0.0269	0.9545, 0.9545

From *Table 3*, it can be seen that after aggregation, the unified reliability is crisper in nature for both systems. This is also depicted in *Fig. 5*.

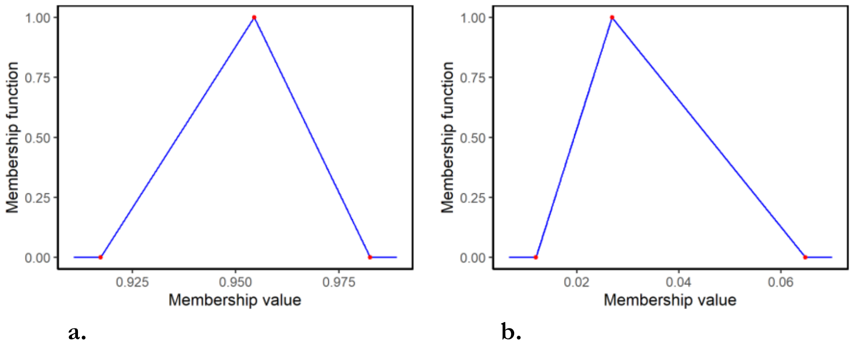


Fig. 5. Reliabilities of series and parallel systems after the aggregation operator; a. series system reliability, b. parallel system reliability.

6 | Results and Discussion

In this paper, the fuzziness of one parameter of the Pareto type I life distribution has been studied using the THFSs approach. The main objective of this study is to assess the level of uncertainty associated with different reliability factors, such as reliability and hazard functions. This is accomplished by measuring the imprecision in the shape parameter into a triangular hesitant fuzzy number. After fuzzifying the parameter, hesitant reliability and hesitant hazard functions for different values of α are displayed in *Fig.1* and *Fig. 2*, respectively. As observed in *Fig. 1* anf *Fig. 2*, both the reliability and hazard functions exhibit a decreasing trend over time. Furthermore, Roohanizadeh et al. [40] has proposed that reliability curves behave like lower and upper bands, with the degree of fuzziness in the measures being equivalent to the width of these bands. It is also noteworthy from *Fig. 1* and *Fig. 2* that an increase in the value of α leads to a reduction in the bandwidth for each hesitant fuzzy reliability characteristic, resulting in more precise bandwidths being achieved for ϕ_3.

In order to maintain the distinctive triangular shapes seen in *Figs. 3-5*, we employed three triangular hesitant fuzzy numbers as part of our methodology, which was subsequently applied to investigate the reliability of both series and parallel systems. The hesitant fuzzy reliabilities for these systems, at various values of α, were calculated numerically and are presented in *Table 1* and *Table 2*, with graphical representations provided in *Fig. 3* and *Fig. 4*. Notably, *Fig. 3* illustrates that the series system exhibits its narrowest bandwidth for ϕ_3, as detailed in *Table 1*. To further refine the precision of these bandwidths, we applied the WAO to each system, resulting in the aggregation of reliabilities for different α values, which can be found in *Table 3* and visually depicted in *Fig. 5*.

This novel approach to applying an aggregation operator demonstrates its utility in enhancing the reliability of each system. For instance, consider the case of series reliability in *Table 1* at $\alpha = 0$, which initially ranged from 0.0267 to 0.1337. After aggregation, this range narrowed significantly, falling between 0.012 and 0.0648. Hence, this outcome underscores the efficacy of the weighted averaging operator, which not only streamlines the results but also emphasizes the remarkable potential of HFSs.

7 | Conclusion

In this article, the concept of THFS, specially designed for Pareto type I distribution with the help of three triangular hesitant fuzzy numbers ϕ_1, ϕ_2 and ϕ_3 has been developed. The hesitant fuzzy reliability and hazard functions show a downward trend with respect to time, and the most precise result is drawn for the third triangular hesitant fuzzy number ϕ_3. The method is further used to investigate the reliability of series and parallel systems, and the results are unified with the help of a weighted averaging operator. The advantages

of employing the weighted averaging operator become evident in the numerical analysis, where the reliability of each system approaches a crisp set. However, it is worth noting that reducing computational complexity for highly intricate systems remains a primary challenge. Future research endeavors could extend this study by exploring the use of various fuzzy sets, including dual HFSs, for the assessment of reliability in complex systems.

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Author Contribution

Conceptualization, A.K.; Methodology, A.K., and W.C.; Validation, W.C, and M.A.; Analysis, A.K, and R.M.; Writing-creating the initial design, A.K., and R.M.; Writing-reviewing and editing, W.C and M.A.

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Data Availability

The data used in this study are available in the article.

Conflicts of Interest

The authors declare no conflict of interest.

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